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Question Paper Code : BA1201

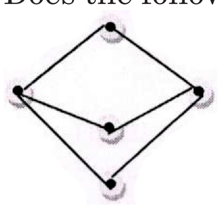
M.C.A. DEGREE EXAMINATION, AUGUST/SEPTEMBER – 2020
REEXAMINATIONS AND FEBRUARY/MARCH – 2021 EXAMINATIONS
First Semester
DMC 5101 – MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE
(Regulations 2018)

Time : Three Hours   Maximum : 100 Marks

Answer ALL questions

PART – A   **(10×2=20 Marks)**

1. Construct the truth table for the statement : $P \to \sim Q$.
2. Explain any two rules of inferences.
3. Using Pigeon hole principle find the minimum number of students to be in a class so that at least two of them born in the same month.
4. How many different number of permutations can be made out of the letters of a word "COMPUTER" ?
5. Which is the smallest non-abelian group ? Write its elements with binary operation.
6. Define a ring and give an example.
7. Does the following diagram represent a Lattice ? Justify.



8. Give an example of modular lattice which is not distributive.
9. State Kleene's theorem.
10. Describe in words the strings in the regular set 1*0.

PART – B   **(5×13=65 Marks)**

11. a) i) Show that $((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$ is a tautology using equivalences. **(7)**

ii) Show that $(\forall x)\ (P(x) \to Q(x)),\ (\exists y)\ P(y) \Rightarrow (\exists x)\ Q(x)$. **(6)**

(OR)

b) i) Obtain PCNF and PDNF of the formula $(P \vee \sim Q) \wedge (P \vee R)$. **(7)**

ii) Show that S is a valid conclusion from the premises $P \rightarrow \sim Q$, $Q \vee R$, $\sim S \rightarrow P$ and $\sim R$. **(6)**

12. a) i) Using mathematical induction prove that $n < 2^n$ for all $n \geq 1$. **(7)**

ii) Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3^n$, $n \geq 0$. **(6)**

(OR)

b) i) A bit is either 0 or 1. A byte is a sequence of 8 bits. Find the number of bytes. Among these how many are (i) starting with 11 and ending with 00 (ii) starting with 11 but not ending with 00 or not starting with 00 but ending with 11 ? **(7)**

ii) There are 2500 students in a college, of these 1700 have taken a course in C, 1000 have taken a course in Pascal and 550 have taken a course in Networking. Further 750 have taken courses in both C and Pascal, 400 have taken courses in both Pascal and Networking and 275 taken courses in both C and Networking. If 200 of these student have taken all the three courses, how many of these 2500 students have not taken in any of these three courses ? **(6)**

13. a) i) Prove that $Z_4 = \{[0], [1], [2], [3]\}$ is a group under operation addition modulo 4. **(7)**

ii) Prove that Kernel of a homomorphism from a group into itself is a normal subgroup. **(6)**

(OR)

b) i) State and Prove Lagrange's theorem. **(7)**

ii) Let $(G, *)$ be a group and a is an element in G. Then prove that the function $f : (G, *) \rightarrow (G, *)$ defined by $f(x) = a * x * a^{-1}$ is an isomorphism. **(6)**

14. a) i) Show that $(N, \leq)$ is a partially ordered set where N is the set of all positive integers and $\leq$ defined by $m \leq n$ if and only if $n - m$ is a positive integer. **(7)**

ii) Show that complement of an element is unique in a complemented distributive lattice. **(6)**

(OR)

b) i) $S = \{1, 2, 3\}$ Draw the Hasse diagram of $(P(S), \subseteq)$. **(7)**

ii) In a Boolean algebra show that $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$. **(6)**

15. a) i) Give a Phase structure grammar that generates the language with strings $\{0^n 1^n 2^n : n = 0, 1, 2, 3, \ldots\}$. **(7)**

ii) Determine whether the string 11101 is in each of the regular sets (i) $\{0, 1\}^*$ (ii) $\{1\}^* \{0\}^* \{1\}^*$ (iii) $\{11\}^* \{01\}^*$. **(6)**

(OR)

b) i) Construct a deterministic finite state automata that recognize each of the following languages with **(7)**

i) the set of strings that begin with two 0's

ii) the set of bit strings that contain two consecutive 0's

iii) the set of bit strings that doesn't contain two consecutive 0's

ii) Draw the state diagram for the finite state machine with state table. **(6)**

State	**f**		**g**	
	Input		**Input**	
	0	1	0	1
s_0	s_1	s_0	0	1
s_1	s_0	s_2	0	1
s_2	s_1	s_1	0	0

PART – C **(1×15=15 Marks)**

16. a) A car manufacturing unit produces 1 car in the first month, 2 cars is in the second month, 3 cars in the third month and so on. Let X_n denotes the number of cars produced in n^{th} month and P_n denotes cumulative number of cars produced at the end of n^{th} month.

i) form a recurrence relation for X_n.

ii) form a recurrence relation for P_n.

iii) solve equation obtained in (i)

iv) solve equation obtained in (ii)

v) How many cars in cumulative are produced at the end of an year ?

(OR)

b) Explain how the elements of S_3 are obtained from an equilateral triangle as permutations. Prove that S_3 is a non-abelian group under composition of permutations. Also, find the subgroups of S_3.